

Lesson 4
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Linear programming in MATLAB

This lesson gives an introduction to linear programming in MATLAB. Students will learn the MATLAB command `linprog`, how to optimize a function, find the maximum and minimum, defining a matrix in MATLAB and convert systems of equations into matrices.

The students will be able to find the unknown values x and y that maximizes or minimizes the objective function. The command `linprog` implements the simplest algorithm to solve linear programming problems. The systems of inequalities can be written in the form $A \cdot X < b$. The minimum or maximum values of the inequality can be found subject to the objective function.

Science Lesson Plan

Teacher: Megha Sunny

Period: Lesson Plan 4

Date(s): November 5 2012

SETTING THE STAGE	
<u>Essential Question</u>	How to solve problems in MATLAB? In this lesson we will introduce linear programming in MATLAB
<u>Content Objective(s)</u> (Student-friendly)	To understand the MATLAB command “linprog ” and find the maximum and minimum values.
<u>Connection to previous or future lessons</u>	This is the fourth lesson in MATLAB. An introduction to basic MATLAB programming was provided in the first lesson.
<u>Critical Thinking Questions</u>	What is the purpose of learning MATLAB? How to solve problems in MATLAB? How to do Linear programming in MATLAB?
<u>Key Vocabulary</u>	Linprog, minimum,maximum
<u>Materials Needed/Safety</u>	Laptops, MATLAB, Pencil, Paper
ACTIVE INSTRUCTION	
<ul style="list-style-type: none"> • Launch (Engage) 	Students will turn on the laptops and open MATLAB in it. Working with laptops will grab the student’s attention.
<ul style="list-style-type: none"> • Investigation (Explore) 	
TIME FOR REFLECTION	
<ul style="list-style-type: none"> • Summarization (Explain & Extend) 	Students will learn about solving linear programming problems. Students will learn how to convert the systems of inequality functions in the form $AX < b$ and find the minimum or maximum value subject to the objective function.
<ul style="list-style-type: none"> • Assessment (Evaluate) 	Observation Listening Questions

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<ul style="list-style-type: none">• Homework	None

Linear Programming in MATLAB

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Optimization Problem

- Find the value of x and y that maximize or minimize the objective function.

$$\left\{ \begin{array}{l} x + y \leq 8 \\ 2x + y \leq 10 \\ x \geq 0 \\ y \geq 0 \end{array} \right.$$

- Maximum for $f = 100x + 40y$

The command *linprog*

- The command `linprog` implements the simplex algorithm to solve linear programming problems.
- $x = \text{linprog}(f, A, b)$ solves the $\min f'x$ such that $A * x \leq b$
- where matrix A and vector b defines the linear constraints and f defines the objective function.

$A * x \leq b$ form

- The inequality functions can be written as

$$\left\{ \begin{array}{l} x + y \leq 8 \\ 2x + y \leq 10 \\ x \geq 0 \\ y \geq 0 \end{array} \right. \quad \longrightarrow \quad \begin{array}{l} x + y \leq 8 \\ 2x + y \leq 10 \\ -x + 0y \leq 0 \\ 0x - y \leq 0 \end{array}$$

where $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$

and $b = \begin{bmatrix} 8 \\ 10 \\ 0 \\ 0 \end{bmatrix}$

Objective Function

- The objective function $f=100x+40y$ can be written in the form

$$f = \begin{bmatrix} 100 \\ 40 \end{bmatrix}$$

Minimum and Maximum Values

- The minimum value can be found as
 $x_{\min} = \text{linprog}(f, A, b);$
 $[x_{\min}, f_{\min}] = \text{linprog}(f, A, b)$
- The maximum values can be found as
 $x_{\max} = \text{linprog}(-f, A, b);$
 $[x_{\max}, f_{\max}] = \text{linprog}(-f, A, b)$

Linear Programming in MATLAB

1. Suppose you make and sell skin lotion. A quart of regular skin lotion contains $2c$ oil and $1c$ coca butter. A quart of extra rich skin lotion contain $1c$ oil and $2c$ coca butter. You will make a profit of $\$10/\text{qt}$ on regular lotion and a profit of $\$8/\text{qt}$ on extra rich lotion. You have $24c$ oil and $18c$ coca butter.

- i. Write the system of inequalities to model the situation.

- ii. Write the matrix A , b , and f

- iii. How many of quarts of lotion should you make to maximize the profit?

- iv. What is the maximum profit?

2. You are going to make and sell bread. A loaf of Irish soda bread is made with $2c$ flour and $0.25c$ sugar. Kugelhopf cake is made with $4c$ flour and $1c$ sugar. You will make a profit of $\$1.50$ on each loaf of Irish soda bread and a profit of $\$4$ on each cake. You have $16c$ flour and $3c$ sugar.

- i. Write the system of inequalities to model the situation.

ii. Write the matrix A , b , and f

i.

iii. How many of each kind of bread should you make to maximize the profit?

iv. What is the maximum profit?

3. Suppose General Motors makes a profit of \$100 on each Chevrolet, \$200 on each Buick, and \$400 on each Cadillac. These cars get 20, 17, and 14 miles a gallon respectively, and it takes respectively 1, 2, and 3 minutes to assemble one Chevrolet, one Buick, and one Cadillac. Assume the government mandates the company that the average car has a fuel efficiency of at least 18 miles a gallon. Under these constraints, determine the optimal number of cars, maximizing the profit, which can be assembled in one 8-hr day. Give all MATLAB commands and the final result.